ITCS 312/314: Formal Languages

Final Exam, 1st Semester 2013/2014, Form: $\boxed{A}$

Name: ____________________

Student Number: ____________________

Section: ____________________

Section 1. (1 point each)

Mark the following statements with True if they are true and False otherwise.

____ Turing machines are superior in power to finite state machines and NPDAs and can accept any formal language.

____ Every nondeterministic finite automaton M can be converted to a deterministic one M' that accepts the same language.

____ An NPDA can accept any language that is generated by a regular grammar.

____ There exists a Turing machine which accepts the language $L = \{a^n b^m a^{n+m} : n, m \geq 0\}$.

____ The software JFLAP can be used to transform an NFA to a DFA and to find a regular grammar for it.

____ If there exists a word from the language of the grammar such that the word has two derivation trees then the grammar is ambiguous.

____ The programming languages C++ and Java are all examples of context-free languages.

____ The grammar $S \to aSb|bSa|SS|A, A \to aAb|\lambda$ is ambiguous.

____ In order for a NPDA to accept a word w, it must scan all its letters and it must halt in a final state.

____ Given a set S which is countable and infinite then its power set 2^S is uncountable.

____ Any context free grammar for a language that does not contain λ can be generated by a grammar in Chomsky Normal Form.

____ By definition, a function f with domain D is Turing-computable if there exists a Turing machine M such that for every $x \in D$, $q_0 x \vdash^* q_f f(x)$, where $q_f \in F$.

____ In order to use exhaustive search to find a parsing for a word using any given grammar, we only have to eliminate lambda productions from the grammar.

____ Let L_1 and L_2 be two regular languages, then $(L_1^* \cup L_2^*)$ is also regular.

____ The set of all languages over an alphabet Σ is defined as 2^{Σ^*}.

____ We can show that the language $L = \{w \in \{0,1\}^* : n_0(w) \text{ is odd and } n_1(w) \text{ is odd}\}$ is not regular using the pumping lemma.

____ The language $L = \{a^n b^n a^n b^n : n \geq 1\}$ can be accepted by an NPDA.

____ If a language is accepted by a Turing machine then it must be a recursive language.

____ ANTLR can be used to generate lexical analysers and parsers for a given context-free grammar.

____ The language $L(a(b)^* + (aa)^*b^*(aa)^* + bb^*a)$ contains the string $abbaaaaa$.

Section 2. (5 points each)

1. Classify the following language according to the Chomsky hierarchy. Write the name of the smallest class of languages that a language is included in. For example, if the language is regular and context-free, then you should use say it is context-free.

Language	Chomsky Heirarchy Type
$\{a^n b^n c^{2n} : n \geq 0\}$	
$\{a^{2n} b^{2m} : n \geq 0, m \geq 1\}$	
$\{w \in \{0,1\}^* : w$ has an even number of 1's$\}$	
$\{w \in \{a,b\}^* : w$ has more a's than b's $\}$	
Every $w \in \{a,b\}^+$ which do not contain the substring abba	

2. Consider the following finite automaton.

(a) Convert it to a DFA.

(b) What is the language accepted by this automaton?

3. Show that the language

$$L = \{a^n b^\ell : n \geq 2\ell\}.$$

is context free.

4. Consider the following Turing machine: $M = (\{q_0, q_1, q_2\}, \{0, 1\}, \Gamma, \delta, q_0, \Box, \{q_5\})$ where the transition graph is defined as

(a) What is the language accepted by M?

(b) What are the elements of the set Γ in the Turing-machine?

5. Show that the following language is non-regular using the pumping lemma

$$L = \{a^{3^n} : n \geq 0\}$$

6. Construct a Turing machine which will compute the following function $f(m) = m \mod 4$ where m is a positive integer. Note that m is represented on the tape by unary notation.

7. Construct an NPDA for the following language.

$$L = \{w \in \{a,b\}^* : a^{3n}b^n\}$$

8. Find a regular grammar for the following language $L = \{w \in \{a,b\} : \; n_a(w) \text{ is divisible by } 3\}$

9. Find a Turing machine that accepts the language $L = \{a^n b^{2n} c^{3n} : n \geq 1\}$

10. What is the main difference between a recursive and a recursively enumerable languages?

Use the diagonalization method to show that the number of real numbers between 0 and 1 (inclusive) is uncountable.